**Department of Mathematics**

**Pattamundai College, Pattamundai**

# 1st Semester

## Discrete Mathematics

Core - 2

**[ Objective ]**

**Unit - I**

1. What is set - bulider form of a set of prime numbers.
2. Give an example of singleton set.
3. If A = { x : x is a letter of word "MATHEMATICS" } and
   B = x : x is a letter of word COMBINATION} then $A \cong B$ (T/F)
4. Suppose $|A|$ = m and $|B|$ = n then the total number of relation from B to A are _____
5. Every function is a relation. (T/F)
6. The relation from A to B is same as the relation from B to A . (T/F)
7. Suppose $|A|$ = 3 and $|B|$ = 2 then the total number of bijective function is _____.
8. If $f(x) = \frac{x^2 + 3x + 5}{(x-4)(x-1)}$, find domain of f.
9. What is compound statements.
10. What is the P, where p : x = 5 is not a prime.
11. Congroence relation is an equivalence relation. (T/F)
12. 8 and 9 are relative prime (T/F)
13. If n = $2^{2019}$ then find the reminder when it divide by 5.
14. If $a \equiv b$ (mod m) then $a+c \equiv b+c$ (mod m), whene c is any integer. (T/F)

**Unit - II**

15. Write the statement of pigeonhole principle.
16. What is weak indoction.
17. What is the difference between the permutation and combination.
18. What is circular permotation.
19. Write the statement of moltinomial theorem.
20. What is recurrence relations.

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21. What is generating function.

22. $\left(Co_n\right)^2 + \left(n_{c_1}\right)^2 + \left(n_{c_2}\right)^2 + .........+ \left(n_{c_n}\right)^2 =$ ______?

23. $\frac{n_{c_o}}{2} + \frac{n_{c_1}}{3} + \frac{n_{c_2}}{4} + .........+ \frac{n_{c_n}}{n+2} =$ ______ ?

24. Compute the value of $\left(\frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!}\right)$

25. What is principle of inclusion

26. The total number of terms in the binomial expansion.

27. $n_{c_r} = n{-}1_{c_r} + n{-}1_{c_{r-1}}$ (T/F)

**Unit - III**

28. The product of two (n×n) matrices always non-commutatse.

29. What is the difference between minors and cofactors.

30. We can calculate the determinants of any matrix.

31. Suppose A is any (n×n) matrix then the det (A) contains the total number of factors.

32. det (A+B) = det(A) + det(B) (T/F)

33. trace (A+B) = trace (A) + trence (B) . (T/F)

34. Suppose Ax = b is a system of equation, where $b \neq o$, when the system has infinite number of equation.

35. Suppose $\lambda$ is the eigen value of A, then the eigen value of $A^n =$ ______ ?

36. The eigen vector of a matriox is unique. (T/F).

**Unit - IV**

37. What is simple graph.

38. What is self - loop.

39. What is regular graph.

40. What is sub graph.

41. What is path.

42. What is pseudo graph.

43. What is connected graph.

44. What is Hamiltonian path.

45. What is Hamiltonian circuit.

46. What is edge of the graph.

47. What is the vecter of the graph.

48. What is multi graph.
49. What is disconnected graph.
50. What is adjacuent edges.

**Section - B**

**Unit - I**

1. Find the values of x and y if (2x–1, 3y+5) = (1–x, 5)
2. If A and B are two sets then prove that $A \subset B$, $A \times A \subset (A \times B) \cap (B \times A)$
3. Let A = {1, 2, 3 , ....... 14}. Define a relation R from A to A by R = {(x,y) : 3x – y = o, where x, $y \in A$} write down its domain, co-domain and range.
4. Prove that n(n+1) (n+5) is a multiple of 3.
5. Find the truth table of $p \vee q \cong \sim (\sim p \sim q)$
6. Show that the set of Natural number is contable.
7. Show that (Q×Q) is countable.
8. If $ac \equiv bc$ (mod m) and gcd(c,m) = 1 then prove that $a \equiv b$ (modm)
9. Constract the table of $Z_{10}$ or $a \equiv b$ (mod 10)
10. Prove that $a \equiv b$ (mod m) and $c \equiv d$ (mod m) then $a+c \equiv b+d$ (mod m)
11. Write the well-ordering property of integer.
12. Prove that the set of irrational number is un countable.
13. Give an example of a relation which is reflexive, symmetic but not transitive.
14. If p is prime and $ab \equiv 0$ (mod p). Show that either $a \equiv o$ (mod p) or $b \equiv o$ (mod p)

**Unit - II**

15. Prove that every integer $n \geq 2$ can be factor as a product of primes.
16. Suppose a teacher has 20 book but the total number of student is . 15, then show that pigeonhole principle can be apply.
17. Prove that $(4^n+15n-1)$ is divisible by 9 for all n.
18. Find the number of diagonal of a polygon of n sides.
19. Find n when $n+1_{p_4} = 2.\, n_{p_4}$
20. Find the total number of ways in which the letters of the word PRESENTATION can be arranged.
21. A bag contains 5 black and 6 white balls from which 7 balls are drawn. Determine the number of wans in which at least 3 black balls can be drawn.
22. Find the middle term in the expansion of $(3a+9c)^{11}$.
23. In how many ways can 'n' persons sit at a round table instead of sitting in a row ?
24. Find the number and sum of divisons of 215622.
25. If a/bc and gcd (a, b) = 1 then a/c.

P.T.O.

**Unit - III**

26. of A and B are two matrices c is any scalor then prove that c (A+B) = cA + cB.
27. Prove that a non-singular matrix has a always non zero determinants.
28. Prove that if two rows or columns of a determinant are Indentical, then the value of the determinant is zero.
29. Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 7 \\ 0 & 0 & 3 \end{pmatrix}$
30. Show that if each column has one-one piovt then the system of equation has either unique solution or no solution.
31. Show that if each row has one and only one piovt then the system has either unique solution or infinite solution.
32. Show that the eigen value of A and $A^T$ are equal.
33. Show that the eigen vector of A and $A^{-1}$ are equal.
34. If $\lambda$ is an eigen value of the matrix A then prove that $\lambda^2$ is an eigen value of $A^2$.
35. If x is an eigen vector of A corresponding to the eigen value $\lambda$ , then prove that is an eigen vector of $A^n$ corresponding to the eigen value $\lambda^n$ .

**Unit - IV**

36. Prove thet the maximum number of edges in a connected simple graph with n vectices is $\frac{n(n-1)}{2}$
37. Prove that the sum of the degree of all vectices in a graph in twice of the number of edges in the graph.
38. Prove that the number of odd degree vertices in a graph is always even.
39.

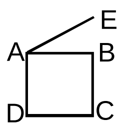



fig - 1

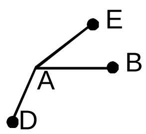
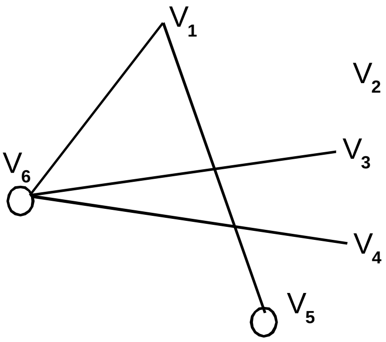



fig - 2

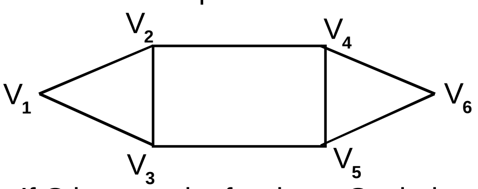
Show that fig - 2 is a sub graph of fig - 1.

40. Prove that when a graph G contains a u - v walk of length k, then G has a u-v path at most of length k.
41. Prove that a graph having exactly two vertices of odd degree must have a path joining these two vertices.
42. Prove that every u-v trail contains u-v path.
43. If G is disconnected graph, then $\overline{G}$ is a connected graph.
44. If G is a graph of order n such that for every two non-adjacent vertices U and V and G.

45. Find the degree of each vectex in the graph given below.




46. Draw the complement of the following graph.




47. If G is a graph of order n, Such that d(v) $> \frac{n-1}{2}$ then for every vertex v of G, it must connected.

48. Draw the fiagram of the following graph G(V,E)

$V = \{ V_1, V_2, V_3, V_4, V_5, V_6 \}$, $E = \{ (V_1,V_4), (V_1,V_5), (V_2, V_3), (V_2,V_6), (V_3,V_5) \}$

49. Let G be a graph with n vertices and e edges then such that G has a vectex of degree k such that $k \geq \frac{2e}{n}$.

50. Justify whether it is possible or not to draw a graph with 12 vertices having 9 edges.

**Section - C**

**Unit - I**

1. Let A = {a, b, c} and the relation is f = {(a,a), (b,b), (c,c), (a,b), (b,a), (a,c), (c,a), (b,c), (c,d)} . Show that f is an equivalence relation.

2. Constract a truth table of $p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$

3. Prove that card (0,1) = card [0,1] = card (R)) = un countable.

4. Use chinese remainder theorem to solve the following question.

$x \equiv 1$ (mod3), $x \equiv 2$ (mod5) and $x \equiv 3$ (mod 7)

5. State and prove Fermat's little theorem.

**Unit - II**

6. Prove it by the method of induction. 1.3 + 2. 4 + 3.5 + ........ + n(n+2) = $\frac{n(n+1)(2n+7)}{6}$

7. State and prove the fundamental therem of Arithmetic.

8. State and prove multinomal theorem.

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9. Find the coefficient of $x^4$ in the expansion of $(1+3x+10x^2)\left(x+\frac{1}{x}\right)^{10}$

10. Prove that $2n_{c_0} + 2n_{c_2} + 2n_{c_3} + \dots\dots + 2n_{c_{2n}} = 2^{2n-1}$

**Unit - III**

11. State and prove the rank and nullity theorem.

12. Find the adjoint and inverse of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix}$

13. Find the row-reduction echelon form of the matrix $A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{pmatrix}$

14. Find the solution of the system of linear equations.
    $2x - 3y = 1,\ 2x - y + z = 2,\ 3x + y - 2z = 1.$

15. Find the eigen value and eigen vector of the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

**Unit - IV**

16. A bipartite graph with n vertices has at must $\left(\frac{n^2}{4}\right)$ edges.

17. Prove that, In a connected graph with n vertices, the minimum number of edges is n–1

18. Prove that the minimum number of edges in a simple graph with ‘n’ vertices is n–k, where k is the number of connected components of the graph.

19. A connected graph is Eulerian if and only if it can be decomposed into cycles.

20. Prove that, the maximum number of edges in a simple graph with n number of vertices and k components will be $\frac{(n-k)(n-k+1)}{2}$.

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